GATE 2027 EC, Section 8

Electromagnetics

Maxwell's equations, boundary conditions, plane waves, reflection, transmission lines and Smith chart, waveguides, optical fibres and antennas.

53 formulas in 10 topics. Equations are numbered section.topic.formula, for example (8.1.1).

8.1 Constants

Free space (8.1.1)

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m; $\mu_0 = 4\pi \times 10^{-7}$ H/m

Speed and impedance (8.1.2)

$c = 1/\sqrt{(\mu_0 \varepsilon_0)} \approx 3 \times 10^8$ m/s; $\eta_0 = \sqrt{(\mu_0/\varepsilon_0)} \approx 120\pi \approx 377\ \Omega$

8.2 Maxwell's equations

Gauss (electric) (8.2.1)

$\nabla \cdot D = \rho_v$; $\oint D \cdot dS = Q_{enc}$

Gauss (magnetic) (8.2.2)

$\nabla \cdot B = 0$; $\oint B \cdot dS = 0$

Faraday (8.2.3)

$\nabla \times E = -\partial B/\partial t$; $\oint E \cdot dl = -d\Phi/dt$

Ampère–Maxwell (8.2.4)

$\nabla \times H = J + \partial D/\partial t$; $\oint H \cdot dl = I_{enc} + I_d$

Constitutive (8.2.5)

$D = \varepsilon E$; $B = \mu H$; $J = \sigma E$

Continuity (8.2.6)

$\nabla \cdot J = -\partial \rho_v/\partial t$

Loss tangent (8.2.7)

$\tan \delta = \sigma/(\omega\varepsilon)$; $\gg 1$ good conductor, $\ll 1$ good dielectric

8.3 Boundary conditions

Electric (8.3.1)

$E_{t1} = E_{t2}$; $D_{n1} - D_{n2} = \rho_s$

Magnetic (8.3.2)

$H_{t1} - H_{t2} = K$ (surface current); $B_{n1} = B_{n2}$

Perfect conductor (8.3.3)

$E_t = 0$ and $B_n = 0$ at the surface

8.4 Statics (quick reference)

Point charge (8.4.1)

$E = Q/(4\pi\varepsilon r^2)$; $V = Q/(4\pi\varepsilon r)$

Capacitance (8.4.2)

Parallel plate $C = \varepsilon A/d$; coaxial $C = 2\pi\varepsilon L/\ln(b/a)$

Energy density (8.4.3)

$w_E = \tfrac{1}{2}\varepsilon E^2$; $w_H = \tfrac{1}{2}\mu H^2$

8.5 Plane waves

Propagation constant (8.5.1)

$\gamma = \alpha + j\beta = \sqrt{(j\omega\mu(\sigma + j\omega\varepsilon))}$

Lossless medium (8.5.2)

$\beta = \omega\sqrt{(\mu\varepsilon)}$; $v = 1/\sqrt{(\mu\varepsilon)}$; $\eta = \sqrt{(\mu/\varepsilon)}$

Good conductor (8.5.3)

$\alpha = \beta = \sqrt{(\pi f \mu \sigma)}$; skin depth $\delta = 1/\sqrt{(\pi f \mu \sigma)}$; $\eta = (1 + j)/(\sigma\delta)$

Wavelength, velocities (8.5.4)

$\lambda = 2\pi/\beta$; $v_p = \omega/\beta$; $v_g = d\omega/d\beta$

Poynting vector (8.5.5)

$S = E \times H$; $\langle S \rangle = \tfrac{1}{2}\mathrm{Re}(E \times H^*)$; plane wave: $E_0^2/(2\eta)$

Polarisation (8.5.6)

Linear: components in phase; circular: equal amplitude, 90° apart; else elliptical

8.6 Reflection and refraction

Normal incidence (8.6.1)

$\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$; $\tau = 2\eta_2/(\eta_2 + \eta_1) = 1 + \Gamma$

Snell's law (8.6.2)

$n_1 \sin\theta_1 = n_2 \sin\theta_2$; $n = \sqrt{(\varepsilon_r \mu_r)}$

Brewster angle (8.6.3)

$\tan\theta_B = n_2/n_1$ (parallel polarisation, non-magnetic)

Critical angle (8.6.4)

$\sin\theta_c = n_2/n_1$ $(n_1 > n_2)$

8.7 Transmission lines

Characteristic impedance (8.7.1)

$Z_0 = \sqrt{((R + j\omega L)/(G + j\omega C))}$; lossless $\sqrt{(L/C)}$

Propagation constant (8.7.2)

$\gamma = \sqrt{((R + j\omega L)(G + j\omega C))}$; lossless $\beta = \omega\sqrt{(LC)}$

Input impedance (lossless) (8.7.3)

$Z_{in} = Z_0(Z_L + jZ_0 \tan \beta l)/(Z_0 + jZ_L \tan \beta l)$

Stubs (8.7.4)

Short: $Z_{in} = jZ_0 \tan \beta l$; open: $Z_{in} = -jZ_0 \cot \beta l$

Special lengths (8.7.5)

$\lambda/4$: $Z_{in} = Z_0^2/Z_L$; $\lambda/2$: $Z_{in} = Z_L$

Reflection (8.7.6)

$\Gamma_L = (Z_L - Z_0)/(Z_L + Z_0)$; $VSWR = (1 + |\Gamma|)/(1 - |\Gamma|)$

Return loss (8.7.7)

$RL = -20 \log|\Gamma|$ dB

S-parameters (8.7.8)

S_{11} = input reflection; $|S_{21}|^2$ = transmission gain; reciprocal $S_{12} = S_{21}$; lossless $S^H S = I$

Smith chart (8.7.9)

One full turn = $\lambda/2$; clockwise = towards generator; voltage max/min are $\lambda/4$ apart

8.8 Waveguides

Rectangular cut-off (8.8.1)

$f_{c,mn} = (c/2)\sqrt{((m/a)^2 + (n/b)^2)}$; dominant TE_{10}: $f_c = c/2a$

Lowest TM mode (8.8.2)

TM_{11} (TM_{m0} and TM_{0n} do not exist)

Guide wavelength (8.8.3)

$\lambda_g = \lambda/\sqrt{(1 - (f_c/f)^2)}$

Velocities (8.8.4)

$v_p = c/\sqrt{(1 - (f_c/f)^2)}$; $v_g = c\sqrt{(1 - (f_c/f)^2)}$; $v_p \cdot v_g = c^2$

Wave impedance (8.8.5)

$Z_{TE} = \eta/\sqrt{(1 - (f_c/f)^2)}$; $Z_{TM} = \eta\sqrt{(1 - (f_c/f)^2)}$

Circular guide (8.8.6)

Dominant TE_{11}: $f_c = 1.841c/(2\pi a)$; TM_{01}: $f_c = 2.405c/(2\pi a)$

8.9 Optical fibres

Numerical aperture (8.9.1)

$NA = \sqrt{(n_1^2 - n_2^2)} = \sin \theta_a$ (from air)

V-number (8.9.2)

$V = (2\pi a/\lambda) \cdot NA$; single-mode if $V < 2.405$

Number of modes (step index) (8.9.3)

$\approx V^2/2$

Modal dispersion (step index) (8.9.4)

$\Delta t \approx Ln_1\Delta/c$, $\Delta = (n_1 - n_2)/n_1$

8.10 Antennas and arrays

Hertzian dipole (8.10.1)

$R_{rad} = 80\pi^2(dl/\lambda)^2$; directivity 1.5

Half-wave dipole (8.10.2)

$R_{rad} \approx 73\ \Omega$; $D = 1.64$ (2.15 dBi)

Quarter-wave monopole (8.10.3)

$R_{rad} \approx 36.5\ \Omega$; $D \approx 3.28$ (over ground)

Directivity, gain (8.10.4)

$D = 4\pi U_{max}/P_{rad}$; $G = \eta_{rad} \cdot D$

Effective aperture (8.10.5)

$A_e = \lambda^2 G/(4\pi)$

Friis transmission (8.10.6)

$P_r = P_t G_t G_r(\lambda/4\pi R)^2$

Far field (8.10.7)

$r > 2D^2/\lambda$

Uniform linear array (8.10.8)

$AF = \sin(N\psi/2)/(N \sin(\psi/2))$, $\psi = \beta d \cos \theta + \alpha$

Broadside / end-fire (8.10.9)

Broadside: $\alpha = 0$; end-fire: $\alpha = -\beta d$

My notes